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An Isospin Analysis of CP Violation in $B_d \rightarrow D\pi, D^*\pi$ and $D\rho$

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Abstract

By use of current experimental data, we carry out an isospin analysis of the weak decays $B \rightarrow D\pi, D^*\pi$ and $D\rho$. It is found that only in $B \rightarrow D\rho$ the strong phase shift of two different isospin amplitudes can be approximately neglected. We derive some useful relations between the CP-violating measurables and the weak and strong transition phases, and illustrate the different effects of final-state interactions on CP violation in $B_d \rightarrow D\pi, D^*\pi$ and $D\rho$.

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In the standard electroweak model, CP violation is naturally described by a non-trivial phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. To test the consistency of the standard model, the most promising way is to study the phenomena of CP violation appearing in weak decays of neutral B mesons [1, 2]. Apart from a variety of B_d decays to CP eigenstates, some exclusive $|\Delta B| = |\Delta C| = 1$ transitions to non-CP eigenstates are also expected to have large CP asymmetries [3, 4]. In most of the previous predictions of branching ratios and CP violation for the latter type of decays, the final-state interactions were either ignored or injudiciously simplified. Hence an improvement of those naive and model-dependent calculations is desirable today, in order to yield reliable numerical results and confront them with the experiments in the near future.

In this note we shall follow a model-independent approach, i.e., the isospin analysis, to study CP violation in the decay modes $B_d \rightarrow D\pi, D^*\pi$ and $D\rho$. An obvious advantage is that the branching ratios for some of these decays have been measured [5], and a full reconstruction of the remaining modes is available at either existing or forthcoming B-meson facilities. By use of the isospin relations and current data, we obtain some constraints on the magnitudes of final-state interactions in the above decays. It is found that only in $B \rightarrow D\rho$ the strong phase shift of two different isospin amplitudes plays an insignificant role. We derive the analytical relations between the CP-violating measurables and the weak and strong transition phases, and illustrate the different effects of final-state interactions on CP violation in $B_d \rightarrow D\pi, D^*\pi$ and $D\rho$.

Let us begin with the decay modes $\bar{B}^0_d \rightarrow D^+\pi^-$, $\bar{B}^0_d \rightarrow D^0\pi^0$, $B^-_u \rightarrow D^0\pi^-$ and their CP-conjugate counterparts. The effective weak Hamiltonians responsible for these processes have the isospin structures $|1, \mp 1\rangle$. After some calculations we obtain the following isospin relations:

$$\begin{aligned} M_{+-} &= \langle D^+\pi^-|\mathcal{H}|\bar{B}^0_d\rangle = V_{cb}V^*_{ud}\left(A_{3/2} + \sqrt{2}A_{1/2}\right) , \\ M_{00} &= \langle D^0\pi^0|\mathcal{H}|\bar{B}^0_d\rangle = V_{cb}V^*_{ud}\left(\sqrt{2}A_{3/2} - A_{1/2}\right) , \\ M_{0-} &= \langle D^0\pi^-|\mathcal{H}|B^-_u\rangle = V_{cb}V^*_{ud}\left(3A_{3/2}\right) ; \end{aligned} \tag{1a}$$

and

$$\begin{aligned} N_{-+} &= \langle D^-\pi^+|\mathcal{H}|B^0_d\rangle = V_{ud}V^*_{cb}\left(A_{3/2} + \sqrt{2}A_{1/2}\right) , \\ N_{00} &= \langle \bar{D}^0\pi^0|\mathcal{H}|B^0_d\rangle = V_{ud}V^*_{cb}\left(\sqrt{2}A_{3/2} - A_{1/2}\right) , \\ N_{0+} &= \langle \bar{D}^0\pi^+|\mathcal{H}|B^+_u\rangle = V_{ud}V^*_{cb}\left(3A_{3/2}\right) . \end{aligned} \tag{1b}$$

Here $A_{3/2}$ and $A_{1/2}$ correspond to the isospin 3/2 and 1/2 amplitudes, whose CKM matrix elements have been factored out and whose Clebsch-Gordan coefficients have been absorbed into the definitions of $A_{3/2}$ and $A_{1/2}$. In obtaining eq. (1), we have assumed that there is no mixture of $B \rightarrow D\pi$ with other channels. It is clear that the above transition amplitudes form two isospin triangles in the complex plane:

$$M_{+-} + \sqrt{2}M_{00} = M_{0-} , \qquad N_{-+} + \sqrt{2}N_{00} = N_{0+} . \tag{2}$$

Of course $|M_{+-}| = |N_{-+}|, |M_{00}| = |N_{00}|$ and $|M_{0-}| = |N_{0+}|$ can be directly determined from measuring the branching ratios of $B \rightarrow D\pi$. Then we are able to extract the unknown quantities $A_{3/2}, A_{1/2}$ and the strong phase shift between them by use of eq. (1) or (2). Denoting $A_{3/2}/A_{1/2} = r e^{i\delta}$, we relate r and δ to the measurables through the following formulas:

$$r = \frac{1}{\sqrt{3\,(R_{+-} + R_{00}) - 1}} \;, \qquad \cos\delta = \frac{r}{2\sqrt{2}}\left[3\,(R_{+-} - 2R_{00}) + 1\right] \;, \tag{3}$$

where $R_{+-} = |M_{+-}/M_{0-}|^2$ and $R_{00} = |M_{00}/M_{0-}|^2$ are two observables independent of the uncertainty of the CKM factors. Since eq. (3) results only from the isospin calculations, it is very useful for a model-independent analysis of the relevant experimental data.

Similarly we can find the same isospin relations as eqs. (1-3) for the decay modes $\bar{B}^0_d \rightarrow D^{*+}\pi^-$, $\bar{B}^0_d \rightarrow D^{*0}\pi^0$, $B^-_u \rightarrow D^{*0}\pi^-$; $\bar{B}^0_d \rightarrow D^+\rho^-$, $\bar{B}^0_d \rightarrow D^0\rho^0$, $B^-_u \rightarrow D^0\rho^-$; and their CP-conjugate processes. The existing data on the above channels can be found in ref. [5]. For the purpose of illustration, here we only calculate R_{+-} and R_{00} by taking the central values or upper bounds of the measured branching ratios. The phase space differences induced by the mass differences $m_{D^0} - m_{D^-}$, $m_{\pi^0} - m_{\pi^-}$ and $m_{\rho^0} - m_{\rho^-}$ are negligible, so is the life time difference $\tau_{B_d} - \tau_{B_u}$ [5]. Our results of R_{+-} and R_{00} are listed in Table 1. Accordingly the lower bounds of the isospin parameters r and $\cos\delta$ can be determined, as also shown in Table 1, with the help of eq. (3). These results are consistent with those obtained from the maximum likelihood method in ref. [6].

Decay modes	R_{+-}	R_{00}	r	$\cos\delta$
$B \rightarrow D\pi$	0.566	< 0.090	> 1.02	> 0.78
$B \rightarrow D^*\pi$	0.500	< 0.186	> 0.97	> 0.47
$B \rightarrow D\rho$	0.582	< 0.041	> 1.07	> 0.94

Table 1: Estimates of the decay rate ratios (R_{+-} and R_{00}) and the isospin parameters (r and δ) for the decay modes $B \rightarrow D\pi, D^*\pi$ and $D\rho$.

From Table 1 we observe that the magnitudes of $A_{3/2}$ and $A_{1/2}$ are comparable in all three cases. The current data on $B \rightarrow D\rho$ imply that $\cos\delta_{D\rho}$ is close to unity, i.e., there are not significant final-state interactions in this type of decays. In contrast, the effects of final-state interactions on $B \rightarrow D\pi$ and in particular on $B \rightarrow D^*\pi$ are non-negligible, unless the branching ratios of $\bar{B}^0_d \rightarrow D^0\pi$ and $D^{*0}\pi^0$ are extremely suppressed. Since $\cos\delta \leq 1$, the lower bound of R_{00} can be found from eq. (3) by fixing the value of R_{+-}:

$$R_{00} \geq \frac{1}{2}\left(1 - \sqrt{R_{+-}}\right)^2 . \tag{4}$$

Specifically, we have $R_{00}(D\pi) \geq 0.031$, $R_{00}(D^*\pi) \geq 0.043$ and $R_{00}(D\rho) \geq 0.028$. In Fig. 1 we plot the allowed ranges of r and $\cos\delta$ as the functions of R_{00}. One can see that the smaller δ is, the smaller R_{00} will be. Among the three groups of decay modes under discussion, $B \rightarrow D\rho$ should be the best candidate for testing the factorization approximation and studying the CP asymmetries. Note that the lower bounds of R_{00} obtained above may model-independently isolate the branching ratios of $\bar{B}^0_d \rightarrow D^0\pi^0, D^{*0}\pi^0$ and $D^0\rho^0$. By use of the central values of the available data [5], we get $\mathrm{Br}(\bar{B}^0_d \rightarrow D^0\pi^0) \geq 1.6\times 10^{-4}$, $\mathrm{Br}(\bar{B}^0_d \rightarrow D^{*0}\pi^0) \geq 2.2\times 10^{-4}$ and $\mathrm{Br}(\bar{B}^0_d \rightarrow D^0\rho^0) \geq 3.8 \times 10^{-4}$. It is expected that these three modes can soon be established in experiments.

We proceed to discuss CP violation in the decay modes $B_d \rightarrow D\pi, D^*\pi$ and $D\rho$, where every final state is common to both B^0_d and $\bar{B}^0_d$ mesons. The CP asymmetry is induced by the interplay of decay and $B^0_d - \bar{B}^0_d$ mixing [1, 2]. Taking $B_d \rightarrow D\pi$ for example, we define the CP-violating interference term via $B^0_d \rightarrow D^+\pi^-$ $(D^0\pi^0)$ versus $\bar{B}^0_d \rightarrow D^+\pi^-$ $(D^0\pi^0)$ as ξ_{+-} (ξ_{00}); and that via $B^0_d \rightarrow D^-\pi^+$ $(\bar{D}^0\pi^0)$ versus $\bar{B}^0_d \rightarrow D^-\pi^+$ $(\bar{D}^0\pi^0)$ as ζ_{-+} (ζ_{00}). Explicitly, these measurables are expressed as

$$\xi_{+-} = \mathrm{Im}\left(\frac{V_{td}V^*_{tb}}{V_{tb}V^*_{td}} \cdot \frac{M_{+-}}{\tilde{M}_{+-}}\right) , \qquad \xi_{00} = \mathrm{Im}\left(\frac{V_{td}V^*_{tb}}{V_{tb}V^*_{td}} \cdot \frac{M_{00}}{\tilde{M}_{00}}\right) , \tag{5a}$$

$$\zeta_{-+} = \mathrm{Im}\left(\frac{V_{tb}V^*_{td}}{V_{td}V^*_{tb}} \cdot \frac{N_{-+}}{\tilde{N}_{-+}}\right) , \qquad \zeta_{00} = \mathrm{Im}\left(\frac{V_{tb}V^*_{td}}{V_{td}V^*_{tb}} \cdot \frac{N_{00}}{\tilde{N}_{00}}\right) , \tag{5b}$$

where the decay amplitudes $\tilde{M}$ and $\tilde{N}$ are given by [7]

$$\begin{aligned} \tilde{M}_{+-} &= \langle D^+\pi^-|\mathcal{H}|B^0_d\rangle = V_{cd}V^*_{ub}\left(\tilde{A}_{3/2} - \sqrt{2}\tilde{A}_{1/2}\right) , \\ \tilde{M}_{00} &= \langle D^0\pi^0|\mathcal{H}|B^0_d\rangle = V_{cd}V^*_{ub}\left(\sqrt{2}\tilde{A}_{3/2} + \tilde{A}_{1/2}\right) ; \end{aligned} \tag{6a}$$

and

$$\begin{aligned} \tilde{N}_{-+} &= \langle D^-\pi^+|\mathcal{H}|\bar{B}^0_d\rangle = V_{ub}V^*_{cd}\left(\tilde{A}_{3/2} - \sqrt{2}\tilde{A}_{1/2}\right) , \\ \tilde{N}_{00} &= \langle \bar{D}^0\pi^0|\mathcal{H}|\bar{B}^0_d\rangle = V_{ub}V^*_{cd}\left(\sqrt{2}\tilde{A}_{3/2} + \tilde{A}_{1/2}\right) . \end{aligned} \tag{6b}$$

For simplicity, we denote $\tilde{A}_{3/2}/\tilde{A}_{1/2} = \tilde{r}e^{\mathrm{i}\delta}$, where δ is the same as the strong phase shift in eqs. (1) and (3). Subsequently we use the Wolfenstein parameters [8] and the angles of the unitarity triangle [5] to express the CKM matrix elements. To a good degree of accuracy, we have $V_{ud} \approx V_{tb} \approx 1$, $V_{cd} \approx -\lambda$, $V_{cb} \approx A\lambda^2$, $V_{ub} \approx A\lambda^3\sqrt{\rho^2+\eta^2}e^{-\mathrm{i}\gamma}$ and $V_{td} \approx A\lambda^3\sqrt{(1-\rho)^2+\eta^2}e^{-\mathrm{i}\beta}$. Note that the parameters (β,γ) are dependent upon (ρ,η) through $\tan\beta = \eta/(1-\rho)$ and $\tan\gamma = \eta/\rho$. The CP-violating terms ξ_{+-} (ζ_{-+}) and ξ_{00} (ζ_{00}) turn out to be:

$$\xi_{+-} = \frac{(r\tilde{r}-2)\sin(2\beta+\gamma) + \sqrt{2}r\sin[\delta-(2\beta+\gamma)] + \sqrt{2}\tilde{r}\sin[\delta+(2\beta+\gamma)]}{\lambda^2\sqrt{\rho^2+\eta^2}\ h\left(\tilde{r}^2+2-2\sqrt{2}\tilde{r}\cos\delta\right)} , \tag{7a}$$

$$\xi_{00} = \frac{(2r\tilde{r}-1)\sin(2\beta+\gamma) - \sqrt{2}r\sin[\delta-(2\beta+\gamma)] - \sqrt{2}\tilde{r}\sin[\delta+(2\beta+\gamma)]}{\lambda^2\sqrt{\rho^2+\eta^2}\ h\left(2\tilde{r}^2+1+2\sqrt{2}\tilde{r}\cos\delta\right)} ; \tag{7b}$$

and

$$\zeta_{-+} \;=\; \frac{(2-r\tilde{r})\sin(2\beta+\gamma)+\sqrt{2}r\sin[\delta+(2\beta+\gamma)]+\sqrt{2}\tilde{r}\sin[\delta-(2\beta+\gamma)]}{\lambda^2\sqrt{\rho^2+\eta^2}\;h\left(\tilde{r}^2+2-2\sqrt{2}\tilde{r}\cos\delta\right)}\;, \tag{8a}$$

$$\zeta_{00} \;=\; \frac{(1-2r\tilde{r})\sin(2\beta+\gamma)-\sqrt{2}r\sin[\delta+(2\beta+\gamma)]-\sqrt{2}\tilde{r}\sin[\delta-(2\beta+\gamma)]}{\lambda^2\sqrt{\rho^2+\eta^2}\;h\left(2\tilde{r}^2+1+2\sqrt{2}\tilde{r}\cos\delta\right)}\;, \tag{8b}$$

where h denotes the ratio $\tilde{A}_{1/2}/A_{1/2}$. It should be noted that ζ_{-+} (ζ_{00}) can be obtained from ξ_{+-} (ξ_{00}) by the replacements $\beta \rightarrow -\beta$ and $\gamma \rightarrow -\gamma$. From eqs. (7) and (8) one can see that in general the strong phase shift δ enters the CP asymmetries and plays a significant role. Only when δ is vanishingly small, its effect on ξ_{+-} (ζ_{-+}) and ξ_{00} (ζ_{00}) can be safely neglected. In this case, we have

$$\xi_{+-} \;=\; -\zeta_{-+} \;=\; \frac{r+\sqrt{2}}{\tilde{r}-\sqrt{2}}\cdot\frac{\sin(2\beta+\gamma)}{\lambda^2\sqrt{\rho^2+\eta^2}\;h}\;, \tag{9a}$$

$$\xi_{00} \;=\; -\zeta_{00} \;=\; \frac{\sqrt{2}r-1}{\sqrt{2}\tilde{r}+1}\cdot\frac{\sin(2\beta+\gamma)}{\lambda^2\sqrt{\rho^2+\eta^2}\;h}\;. \tag{9b}$$

Indeed the conditions $\xi_{+-} = -\zeta_{-+}$ and $\xi_{00} = -\zeta_{00}$ were injudiciously taken in most of the previous works (see, e.g., refs. [3, 4]). Considering the isospin results of r and δ given in Table 1 and Fig. 1, we know that only $\cos\delta_{D\rho} \approx 1$ is an accetable approximation to the current data. Thus the CP asymmetries in $B_d \rightarrow D\rho$ are dominated by the $\sin(2\beta+\gamma)$ term of ξ_{+-} (ζ_{-+}) or ξ_{00} (ζ_{00}). In contrast, the previous numerical predictions of CP violation in $B_d \rightarrow D\pi$ and $D^*\pi$ are questionable.

Now let us illustrate the effects of nonvanishing δ on CP asymmetries in $B_d \rightarrow D\pi, D^*\pi$ and $D\rho$. Typically we take $\lambda \approx 0.22$, $\rho \approx -0.07$ and $\eta \approx 0.38$ [9], and this corresponds to $\beta \approx 19.6^0$ and $\gamma \approx 100.4^0$. Fixing R_{+-}, we still use the constraints on R_{00} obtained in Table 1, eq. (4) and Fig. 1. Since there is not any experimental information on the magnitudes of $\tilde{A}_{3/2}$ and $\tilde{A}_{1/2}$, here we make a likely but unjustified approximation: $h \approx 1$ and $\tilde{r} \approx r$, just for the purpose of simplicity and illustration. The changes of ξ_{+-}, ζ_{-+} and ξ_{00}, ζ_{00} with the allowed values of $\cos\delta$ are shown in Figs. 2 and 3 respectively. One can see that in $B_d \rightarrow D\pi$ and $D^*\pi$ the effects of δ are significant, and the approximation $\xi_{+-} = -\zeta_{-+}$ or $\xi_{00} = -\zeta_{00}$ is invalid to a large extent. In comparison, the final-state interactions in $B_d \rightarrow D\rho$ may be negligible, if the decay rate of $\bar{B}^0_d \rightarrow D^0\rho^0$ is further suppressed to allow for $\cos\delta_{D\rho} \rightarrow 1$. Although the strong phase shift δ plays a non-negligible role in each of the above channels, its effect can be well isolated after the measurements of $\bar{B}^0_d \rightarrow D^0\pi^0, D^{*0}\pi^0$ and $D^0\rho^0$. Thus the determination of CP violation in $B_d \rightarrow D\pi, D^*\pi$ and $D\rho$ remains promising in the near future.

The experimental scenarios for observing CP violation in the exclusive $|\Delta B| = |\Delta C| = 1$ transitions have been discussed in the literature (see, e.g., ref. [2]). The basic signal for

CP violation between a channel (e.g., $B^0_d \rightarrow D^+\pi^-$) and its CP-conjugate counterpart ($\bar{B}^0_d \rightarrow D^-\pi^+$) is a nonvanishing ratio of the difference to the sum of their decay rates. For either coherent or incoherent $B^0_d\bar{B}^0_d$ decays to the non-CP eigenstates under study, the CP asymmetries are always proportional to $(\xi_{+-} - \zeta_{-+})$ or $(\xi_{00} - \zeta_{00})$, as shown in ref. [10]. Both the time-integrated and time-dependent measurements are available to establish the CP-violating signals in $B_d \rightarrow D\pi, D^*\pi$ or $D\rho$, after 10^{7-8} $B^0_d\bar{B}^0_d$ events have been accumulated.

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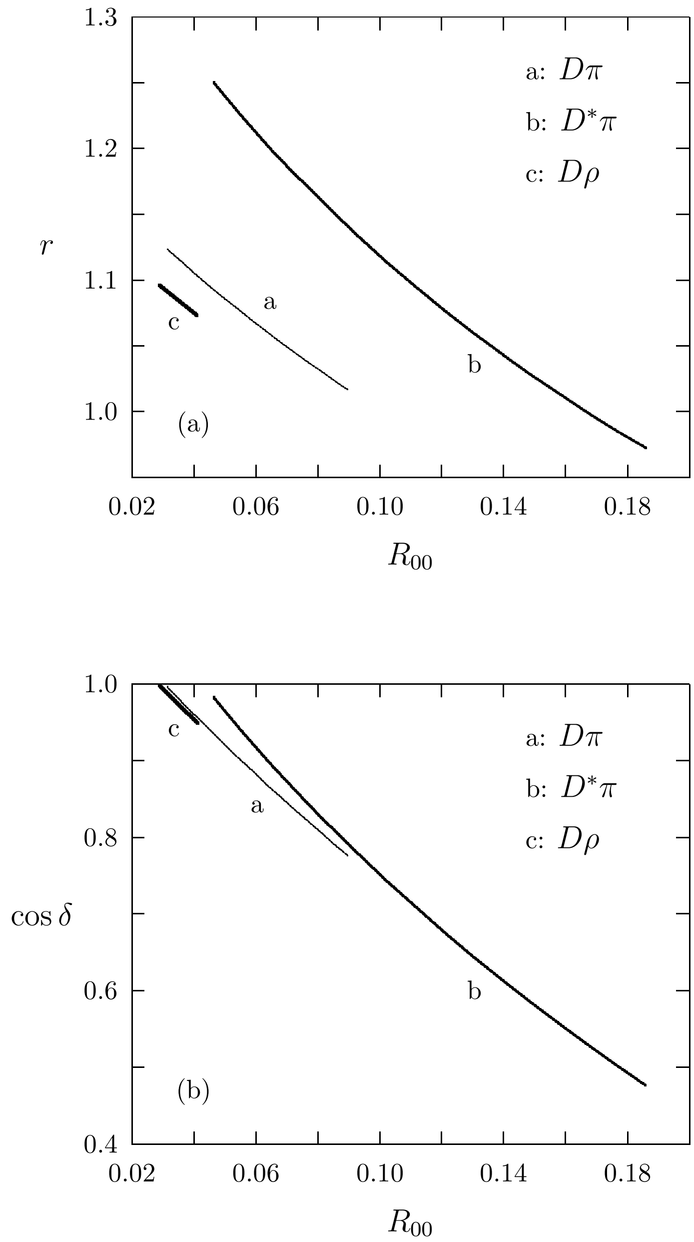

Figure 1: Possible ranges of the isospin parameters $r = |A_{3/2}/A_{1/2}|$ and $\delta = \arg(A_{3/2}/A_{1/2})$ allowed by the current data on $B \to D\pi, D^*\pi$ and $D\rho$.

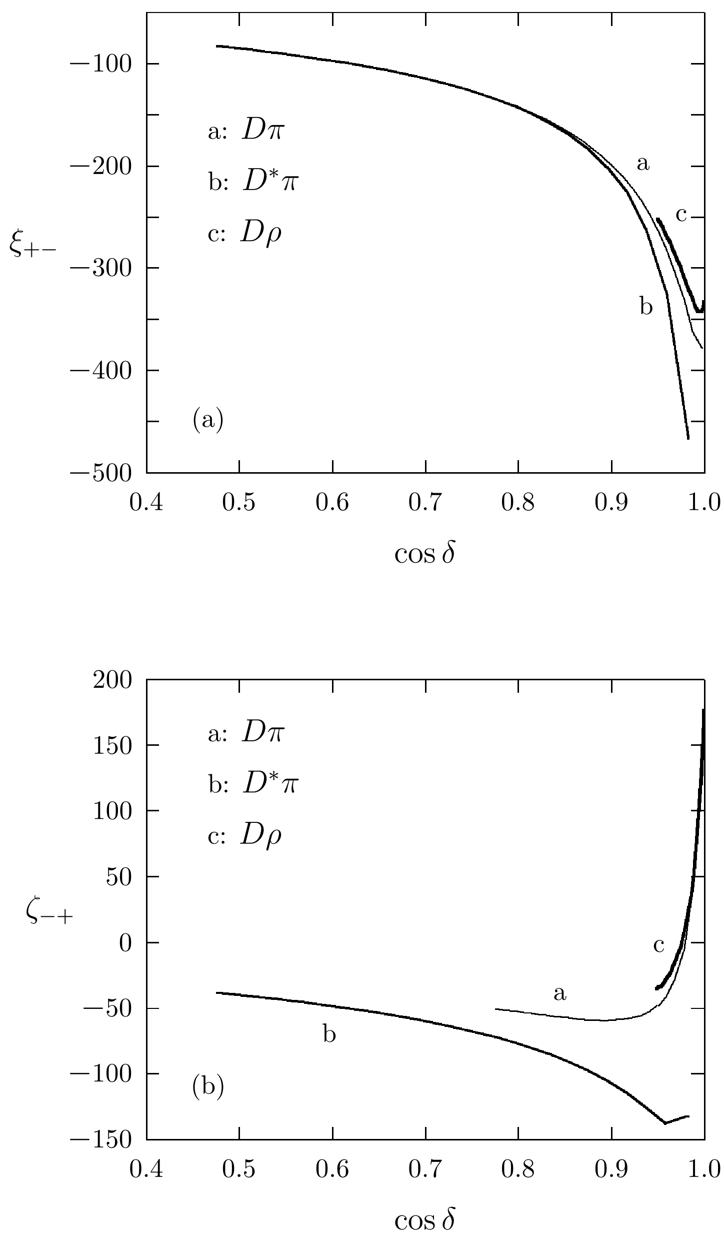

Figure 2: Illustration of the changes of the CP-violating terms ξ_{+-} and ζ_{-+} with the allowed values of the strong phase shift δ in the decay modes $B_d \to D\pi, D^*\pi$ and $D\rho$.

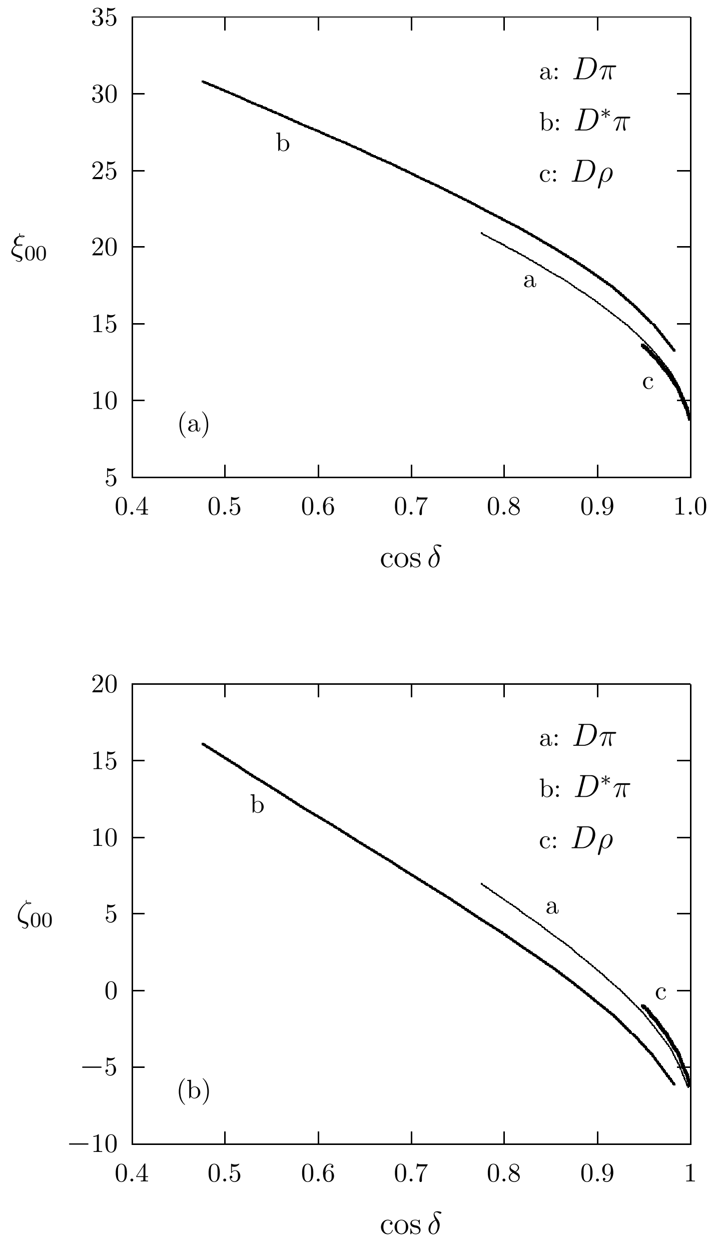

Figure 3: Illustration of the changes of the CP-violating terms ξ_{00} and ζ_{00} with the allowed values of the strong phase shift δ in the decay modes $B_d \to D\pi, D^*\pi$ and $D\rho$.